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A SYSTEM FOR DETERMINING THE PARAMETERS OF THE MOTION OF A BODY IN SPACE

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1. Basic conditions. Independent determination of the velocity and acceleration coordinates, moving in space, is a quite complex problem.

As is known, during the motion of any vehicle, when there are no field or gravitational forces acting upon it, bodies, which are located within the vehicle, are found in a weightless state as a consequence of which, systems, possessing a period of stability, lose their basic quality which results in the correction of the platform's condition (of the vertical) for either the first or second integral of accelerations.

Therefore, determination of the coordinates is the suggested solution of the problem of determining the velocity and acceleration coordinates, for example, for the three angles between the straight lines, which connect the vehicle at the time of its movement with three planets (the coordinates of which are known, naturally), or with the sun and two planets. Differentiating in such a manner the obtained readings of the coordinates for the first time, it is possible to determine the velocity, and differentiating these readings for the second time -- the velocities of the flying vehicle.

However, any system is conceivable which employs the properties of a system corrected in accordance to the second integral of the measured accelerations. The substance of this system consists in the following. Let us have a three-gyroscope power system, which is immobile with regard to stellar space. For gyroscopes of a sufficiently high quality, the "immobility" of the gyro-system may be provided in the conventional manner by two optical systems which are aimed at any two stars. Two platforms will be connected with this gyro-system which have two degrees of freedom each with reference to the gyro-system. The two angles of shift of the first platform with reference to the gyro-system will be proportional to the double integral of the accelerations, measured at two mutually perpendicular directions by accelerometers, which are located on the first platform.

The double integral of acceleration, measured by the third accelerometer, which is on the second platform and perpendicular with the first two, will determine the position of the second platform. In this case, the double integrals will give us the passage route in inter-planetary space, but the first integrals -- of velocity, one can conditionally ignore as the effect of accelerations, which are created by the gravitational fields of the sun and the planets as well as by the instrumental errors of the system. Of course, errors due to the effect of the gravitational fields and of instrumental defects of the system's elements (of the accelerometers and integrators) can achieve substantial values. The correction itself, which is applicable in similar gyro-systems[1], is not applicable here as a result of the weightlessness of the sensitive elements of the accelerometers. ([1]As gyro-systems, we will imply systems which are intended for vehicles moving with relatively low velocities along the surface of the earth.)

The following device may be employed for elimination of the indicated errors. Let us have two optical systems directed at two planets or at the sun and a planet. The angles of shift of the first platform must be proportional to the sum of the double integrals of the velocities and of the deviation angles (measured respectively in two mutually perpendicular planes) of the platform of the first optical system. In such a manner, these angles of deviation become equivalent to the signals of the accelerometers in the geosystem obtained for the deviation of the measurement direction of velocities for a position which is perpendicular to the local vertical (or more precisely, for a position which is perpendicular to the local direction of the lines of force of the gravitational field).

The position of the second platform will be determined by the double integral of the velocities and deviation angles of the platform from the second optical system (which is directed at an angle close to $1/2\pi$ for the first system).

In a plan of such type, the period of oscillations of the platform will depend only on the factors of proportionality which connect the angles of shift of the platforms from the optical systems and the double integrals of their angles of deviation.

It is natural that the specified plan also depends upon the system of coordinates in accordance to which the position of the vehicle within the solar system is determined.

Below, as an example, will be examined a spherical system of coordinates with the origin at the center of the earth and with an original plane which coincides with the ecliptic plane and a prime meridian which passes through the point of the vernal equinox.

However, other systems may also obviously be selected, for example, with the origin at the center of the earth and an original plane which coincides with the plane of the celestial equator, the orthodromic system, a system of rectangular coordinates, and so forth.

Obviously, for automatic control, it is expedient to apply a system of coordinates with the center at the point of motion assignment.

It should be noted that inasmuch as all planets are situated close to the ecliptic plane, then the accepted system of coordinates is very close to orthodromic.

Upon utilization of the first of the indicated coordinate system, the vehicle's positional coordinates will be φ and α, which are formed by the radius-vector of the vehicle with the initial plane (angle φ) (Fig. 1) and by the angle between the prime meridian and the vehicle's positional meridian (angle α). The coefficient r of the vehicle's radius-vector is the third coordinate. In the considered case, such a position of the first platform will be most simple when its plane is perpendicular to the radius-vector (however, such a disposition is not the only one possible). Its angles of shift will be φ and α with regard to the gyro-system. The position of the platform, relative to the radius-vector, obviously must be such that the measurement axis of the two accelerometers, which are connected with the platform, would coincide with the meridian and the parallel of the selected coordinate system. Accordingly, also the angles of deviation from the radius-vector $\Delta \varphi$ and β, which is determined by the first optical system, must lie on a plane of the meridian and be perpendicular to its plane, which passes through the origin of the coordinate, that is, in the plane of angles β.

The most simple (but not the only one possible) orientation of the second platform must be such that the axis of measurement of the third accelerometer would coincide with the direction of the radius-vector, that is, with the axis of the first optical system which is oriented towards the sun (or a planet). The position itself relative to the radius-vector depends upon the direction of the second optical system. The angle between the second optical system and the plane of the second platform, obviously, is equal to the angle between the directions to the sun and a planet (or to the angle between the directions towards two planets). Upon movement along the radius-vector and with the passage of time, this angle will change and correspondingly, the angle between the optical system, directed to a planet and the angle of the second platform, must change. Deviations of the second platform from the assigned (computed) position relative to the second optical system also must be integrated twice and the second platform must be corrected in accordance to the signal obtained after double integration.

Let us now determine the advantages and deficiencies of the considered system in comparison with the system indicated at the start. Its deficiency is that instead of three optical systems, which are required for the first system, four are necessary for the considered system. Its advantage is the following. The velocity values of the vehicle's movement, obtained in accordance to the

first plan, will be determined as was indicated as a result of differentiation of the readings of the coordinates. It is easy to see that variations of errors of the coordinates during differentiation will yield, generally speaking, larger values of errors of velocity readings of the moving vehicle. In addition, the more rapidly the errors of coordinate readings change, the greater will be the errors of velocity readings. Upon utilization of the considered plan, sighting errors of the optical systems (which corresponds to errors of coordinates in the first plan) can produce only oscillations of the platforms with an amplitude which is equal to the magnitude of the error and with the period which depends upon the proportionality factor between the double integral of the platform's deviation from the direction towards the heavenly body and the angle of shift of the platform. This period may be selected sufficiently large so that the magnitude of error of the velocity of the moving vehicle may be made significantly less than the velocity error which is obtained in the first plan. This is also natural since it is known that during differentiation, errors increase, but during integration, they are equalized. It is obvious that the aforesaid is in a still greater degree related to the measurement of velocities. If in the first plan, acceleration is obtained as a result of double differentiation, then in the second, it is obtained as a result of direct measurement. (Here, acceleration is not taken into account which appears as a result of the effect of the gravitational forces of the sun and of the planets.

Besides this, one should note that damping of the described system does not produce errors which occur during damping of the geosystem. Damping may be obtained if the shift of the platform will be in addition proportional to the single integral of its deviation angles from the optical system.

2. Equations of the motion of platforms. Although the position of the first platform with regard to the immobile gyro-system will be determined by angles φ and α, and inasmuch as correction of the platform will take place in the plane of angle φ and β, then we will also set up the equations of the motion of the platform for angles φ and β.

The angle φ will be determined from the equation $\varphi = \varphi_1 + \varphi_0$ where φ_0 - is the initial angle between the first platform and the gyro-system which depends upon the initial coordinates of the vehicle. If the radius-vector of the vehicle at the initial moment lies in the ecliptic plane, then $\varphi_0 = 0$.

The values of angles φ, φ_0, and φ_1 are easily perceived in Figure 2.

Obviously, the variations of angle φ_1, which can be performed by special devices, must be proportional to the second integrals for velocities measured with accelerometers, that is,

$$\Delta\varphi_1 = \gamma_\varphi \int_0^t\int S_{j_2}\, dt\, dt$$

where r_φ - is the factor of proportionality, Σj_z - is the sum of all velocities, which are measured with an accelerometer along the z-axis (Figure 1).

On the other hand, from Figure 3 it is apparent that $\Delta\varphi_1 = \varphi_1 - \Delta\varphi$ where $\Delta\varphi$ - is the deviation of the first platform from the computed, in particular from the perpendicular to the radius-vector, position.

Since $\varphi_1 = \frac{S_\varphi}{r}$ then $\Delta\varphi_1 = \frac{S_\varphi}{r} - \Delta\varphi$

or
$$\Delta\varphi = \frac{S_\varphi}{r} - r_\varphi \iint_0^t \Sigma j_z \, dt\, dt.$$

Let us interpret the values which lie beneath the sign of the double integral. For this, let us determine the velocities j_x, j_y and j_z which are measured with accelerometers. It is known that

$$\Sigma j_x = V_x' + \omega_y V_z - \omega_z V_y$$
$$\Sigma j_y = V_y' + \omega_z V_x - \omega_x V_z$$
$$\Sigma j_z = V_z' + \omega_x V_y - \omega_y V_x$$

where V_x', V_y' and V_z' -- are the projections of velocities produced by non-field forces along the mobile axis, V_x, V_y and V_z -- are the integrals of these projections, and ω_x, ω_y and ω_z -- are the projections of the complete angular velocity of the mobile system of coordinates along the mobile axes.

Keeping in mind that in our case

$$V_x = V_r, \quad V_y = V_\alpha, \quad V_z = V_\varphi, \quad \omega_x = \frac{V_\alpha}{r} tg\varphi,$$
$$\omega_y = -\frac{V_\varphi}{r}, \quad \omega_z = \frac{V_\alpha}{r}$$

we will obtain

$$\Sigma j_x = V_r' - \frac{V_\varphi^2 + V_\alpha^2}{r}$$
$$\Sigma j_y = V_\alpha' + \frac{V_\alpha V_r}{r} - \frac{V_\varphi V_\alpha}{r} tg\varphi$$
$$\Sigma j_z = V_\varphi' + \frac{V_\varphi V_r}{r} + \frac{V_\alpha^2}{r} tg\varphi.$$

Let us substitute the obtained values into equations of platforms' motion. we have

$$\Delta\varphi = \frac{S_\varphi}{r} - r_\varphi \iint_0^t \left(V_\varphi' + \frac{V_\varphi V_r}{r} + \frac{V_\alpha^2}{r} tg\varphi\right) dt\, dt.$$

Let us superpose conditions on the value of the coefficient r_φ. Let $r_\varphi = \frac{1}{r}$.

Then it is easy to see that $$\frac{S_\varphi}{r} = r_\varphi \iint_0^t V_\varphi' \, dt\, dt$$

Hence, the equation will have the form

$$\Delta \varphi = -\gamma_\varphi \int_0^t\!\!\int \left(\frac{V_\varphi V_r}{r} + \frac{V_\alpha^2}{r} tg\varphi \right) dt\,dt \tag{2.1}$$

Proceeding in a similar manner for the plane of angle β, we will obtain the following equation:

$$\beta = -\gamma_\beta \int_0^t\!\!\int \left(\frac{V_\alpha V_r}{r} - \frac{V_\varphi V_\alpha}{r} tg\varphi \right) dt\,dt \tag{2.2}$$

One should note that V_α and V_α' could be designated V_β and V_β' (but $\alpha \neq \beta$ -- these angles lie in different planes).

For the second platform, the equation will be constituted somewhat differently. Actually, one can obtain for the second platform

$$\Delta \psi = \frac{\Delta r \sin\gamma}{\rho} - \gamma_\psi \sin\gamma \int_0^t\!\!\int \Sigma j_x \, dt\,dt$$

where $\Delta\psi$ -- is the angle of shift of the second platform with regard to the radius-vector of the vehicle, and γ -- is the angle between the directions to the sun and a planet (Figure 4) (or between the directions to two planets), and ρ -- is the distance from the vehicle to the planet.

Substituting into this equation the value Σj_x and observing that

$$\frac{\Delta r \sin\gamma}{\rho} = \gamma_\psi \sin\gamma \int_0^t\!\!\int V_r' \, dt\,dt$$

where γ_ψ -- is the factor of proportionality which must be equal to $1/\rho$, we will find

$$\Delta \psi = \gamma_\psi \sin\gamma \int_0^t\!\!\int \frac{V_\varphi^2 + V_\alpha^2}{r} \, dt\,dt \tag{2.3}$$

Now, let the angles $\Delta\varphi$, β and $\Delta\psi$ change not only in proportion to the double integrals of the velocities but also in proportion to the sums of the double and single integrals from the angles themselves, that is, let us have

$$\Delta \varphi = -\gamma_\varphi \int_0^t\!\!\int \left(\frac{V_\varphi V_r}{r} + \frac{V_\alpha^2}{r} tg\varphi \right) dt\,dt - K_2 \int_0^t\!\!\int \Delta\varphi \, dt\,dt - k_1 \int_0^t \Delta\varphi \, dt \tag{2.4}$$

Similarly

$$\beta = -\gamma_\beta \int_0^t\!\!\int \left(\frac{V_\alpha V_r}{r} - \frac{V_\varphi V_\alpha}{r} tg\varphi \right) dt\,dt - k_4 \int_0^t\!\!\int \beta \, dt\,dt - k_3 \int_0^t \beta \, dt \tag{2.5}$$

$$\Delta\psi = \gamma_\psi \sin\gamma \iint_0^t \frac{V_\varphi^2+V_\alpha^2}{r}\,dt\,dt - k_6 \iint_0^t \Delta\psi\,dt\,dt - k_5 \int_0^t \Delta\psi\,dt \qquad (2.6)$$

where γ_3, $k_1 - k_6$ -- are the factors of proportionality whereupon the factor γ_3 must be equal to 1/r, but $k_1 - k_6$ -- are constant values.

Differentiating the equations twice, we will obtain for the first equation

$$\Delta\varphi'' + k_1\Delta\varphi' + k_2\Delta\varphi = -\gamma_\varphi\left(\frac{V_\varphi V_r}{r} + \frac{V_\alpha^2}{r} t_{q\varphi}\right) -$$

$$-2\frac{d\gamma_\varphi}{dt}\int_0^t\left(\frac{V_\varphi V_r}{r} + \frac{V_\alpha^2}{r} t_{q\varphi}\right)dt - \frac{d^2\gamma_\varphi}{dt^2}\iint_0^t\left(\frac{V_\varphi V_r}{r} + \frac{V_\alpha^2}{r} t_{q\varphi}\right)dt\,dt$$

If the first item of the right-hand side -- for motion at a velocity, which is equal to the second cosmic velocity and for r, which is equal to the distance from the earth to the sun -- is a magnitude of the order $3.2\cdot10^{-5}$ $1/\sec^2$, which corresponds to a linear velocity of $4.8\cdot10^{-4}$ m/sec^2, whereupon the second and third items, which generally speaking, increase with the passage of time, will achieve the value of the first item only after motion for 150 and 200 days respectively. At the beginning of the motion itself, the values of the second and third items will be evanescently small.

Similarly, for the angles β and $\Delta\psi$, we will have

$$\beta'' + k_3\beta' + k_4\beta = -\gamma_\beta\left(\frac{V_\alpha V_r}{r} - \frac{V_\varphi V_\alpha}{r} t_{q\varphi}\right) -$$

$$-2\frac{d\gamma_\beta}{dt}\int_0^t\left(\frac{V_\alpha V_r}{r} - \frac{V_\varphi V_\alpha}{r} t_{q\varphi}\right)dt - \frac{d^2\gamma_\beta}{dt^2}\iint_0^t\left(\frac{V_\alpha V_r}{r} - \frac{V_\varphi V_r}{r} t_{q\varphi}\right)dt\,dt$$

$$\Delta\psi'' + k_5\Delta\psi' + k_6\Delta\psi = \gamma_\psi \sin\gamma\,\frac{V_\varphi^2+V_\alpha^2}{r} +$$

$$+2\frac{d(\gamma_\psi\sin\gamma)}{dt}\int_0^t\frac{V_\varphi^2+V_\alpha^2}{r}\,dt + \frac{d^2(\gamma_\psi\sin\gamma)}{dt^2}\iint_0^t\frac{V_\varphi^2+V_\alpha^2}{r}\,dt\,dt$$

3. __The determination of the factors of equations for the motion of platforms__. Let us determine the period of oscillation of the platform and the values of the factors of the equations, proceeding from the necessary accuracy of the velocity readings. If the necessary accuracy must be 0.5% of the velocity of the vehicle's

motion, then the period of oscillations of platforms is easily determined, and consequently also the values of the factors if one assumes that attenuation is semiaperiodic, and the amplitude of the oscillation of a platform does not exceed 10 - 12" and the velocity of the movement is equal to seven cosmic velocities (to the second one). The indicated speed is the average speed for a flight to Mars during a time of great opposition under the condition that the flight time is equal to approximately one month.

It is obvious that $T = \frac{2\pi A r}{\Delta V}$ where T -- is the period of oscillations of a platform, A -- is the amplitude of the oscillations, r -- is the distance from the vehicle to the heavenly body, and ΔV -- is the permissible error of velocity determination.

Since,

$$A = \frac{10}{3438\cdot 60} \approx 4.85\cdot 10^{-5}, \qquad r \approx 1.85\cdot 10^{11}\,\mathrm{m}$$

$$\Delta V = \frac{7\cdot 11.2\cdot 10^{3}\cdot 0.5}{100} \approx 3.92\cdot 10^{2}\ \frac{\mathrm{m}}{\mathrm{sec}}$$

then

$$T = \frac{6.28\cdot 4.85\cdot 10^{-5}\cdot 1.85\cdot 10^{11}}{3.92\cdot 10^{2}} \approx 1.44\cdot 10^{5}\,\mathrm{sec} = 40\ \mathrm{hac}$$

Hence, knowing that the attenuation must be semiaperiodic, that is, that

$$k_1^2 = k_3^2 = k_5^2 = k^2\ \text{odd} = 2k_2 = 2k_4 = 2k_6 = 2k\ \text{even}$$

we will obtain that the frequency of the oscillations is equal to $1/2\ k_o$ or $1/2\ \sqrt{2}\ \sqrt{k_e}$.

Thus,

$$\sqrt{K_E} = \sqrt{2}\ \frac{2\pi}{T} = \frac{2\cdot\sqrt{2}\,\pi}{1.44\cdot 10^{5}} \approx 6.16\cdot 10^{-6}\ \frac{1}{sec}$$

$$K_E = 3.8\cdot 10^{-11}\ 1/sec^2, \qquad K_o = 8.7\cdot 10^{-6}\ 1/sec$$

Hence, it is easy to find the error which is produced by the right-hand portion of equations of the motion of platforms.

One can approximately assume $\alpha' r = \varphi' r = r'$, and $\varphi' = \emptyset$. Then assuming the velocity is equal to $7V_{2k}$, where V_{2k} -- is the second cosmic speed, we will obtain

$$\alpha' r = \varphi' r = r' = V = \frac{7}{\sqrt{3}}\, V_{2K} = 45.3\ \frac{Km}{sec}$$

Further

$$\frac{\dot{r}V^2}{r} = \frac{V^2}{r^2} = 6\cdot 10^{-14}\ 1/sec^2$$

and the error is

$$\delta = \frac{6\cdot 10^{-14}}{3.8\cdot 10^{-11}} \approx 1.58\cdot 10^{-3} \approx 5.4'$$

Hence it follows that the magnitude of this error is impermissibly large and it is necessary to compensate it.

4. <u>Compensation of the methodic errors of a system</u>. For compensation of errors, produced by the right-hand portions of the equations, it is sufficient to compensate the sub-integral expressions in equations (2.1) - (2.3). In this case, not only the first but also the following items of the right-hand portions of equations (2.4) - (2.6) automatically become equal to zero. For compensation of these errors, let us supply a method, the substance of which consists of the following. Knowing $V_{\varphi\,err}$, $V_{\alpha\,err}$ and $V_{r\,err}$, which are the readings of the first integrators, and φ err and rerr which are readings of the second integrators from the corresponding velocities, let us form from these values the functions which are beneath the double integrals in equations (2.1) - (2.3) and let us provide signed signals, which correspond to these functions, to the integrators which are the inverse values of the velocities located beneath the integrals. For a correct value of the readings, compensation will be complete. In the presence of errors in the readings, the results of compensation require supplementary analysis. An analysis of such a type of compensated system with regard to errors will be conducted below.

Since the second and third terms of the right-hand portions of the equations at the beginning of flight are evanescently small, then for analysis of the stability of a system, it is sufficient to take into account the fulfillment, as has already been indicated, of compensation of the first term only. In addition, of course, one assumes ν_φ, ν_β, ν_ψ, and also γ as constant values.

Let us extract in equations (2.4) - (2.6) the factors ν_φ, ν_β, and $\nu_\psi \sin\gamma$ in the right-hand portions as the quantity sign, and let us introduce the compensated terms. Then we will obtain

$$\Delta\varphi = -\nu_\varphi\left[\iint_0^t\left(\frac{V_\varphi V_r}{r} + \frac{V_\alpha^2}{r}\,tg\,\varphi + \frac{K_2}{\nu_\varphi}\Delta\varphi + \frac{K_1}{\nu_\varphi}\frac{d\Delta\varphi}{dt} - \Phi\right)dt\,dt\right]$$

$$\beta = -\nu_\beta\left[\iint_0^t\left(\frac{V_\alpha V_r}{r} - \frac{V_\varphi V_\alpha}{r}\,tg\,\varphi + \frac{K_4}{\nu_\beta}\beta + \frac{K_3}{\nu_\beta}\frac{d\beta}{dt} - B\right)dt\,dt\right] \qquad (4.1)$$

$$\Delta\psi = \nu_\psi \sin\gamma\left[\iint_0^t\left(\frac{V_\varphi^2 + V_\alpha^2}{r} - \frac{K_6}{\nu_\psi\sin\gamma}\Delta\psi - \frac{K_5}{\nu_\psi\sin\gamma}\frac{d\Delta\psi}{dt} - \Psi\right)dt\,dt\right]$$

where $\Phi = \frac{V_{\varphi\,err}V_{r\,err}}{r_{err}} + \frac{V_{\alpha\,err}^2}{r_{err}^2}\,tg\,\varphi_{err}$; $B = \frac{V_{\alpha\,err}V_{r\,err}}{r_{err}} - \frac{V_{\varphi\,err}V_{\alpha\,err}}{r_{err}}\,tg\,\varphi_n$; $\Psi = \frac{V_{\varphi\,err}^2 + V_{\alpha\,err}^2}{r}$

as has already been indicated above, the readings of the system.

Let us designate the brackets in the right-hand portions of equations (4.1) respectively ΔS_φ, ΔS_β and ΔS_ψ. Then instead of equations (4.1) one can write

$$\Delta\varphi = -\nu_\varphi\Delta S_\varphi;\quad \beta = -\nu_\beta\Delta S_\beta;\quad \Delta\psi = \nu_\psi\sin\gamma\,\Delta S_\psi \qquad (4.2)$$

Further

$$\Delta S_\varphi'' = a\,\delta V_\varphi + b\,\delta V_r - c\,\delta r + d\,\delta V_\alpha + e\,\delta\varphi + \frac{k_2}{V_\varphi}\Delta\varphi + \frac{k_1}{V_\varphi}\Delta\varphi'$$

$$\Delta S_\beta'' = -f\,\delta V_\varphi + g\,\delta V_r - h\,\delta r + j\,\delta V_\alpha - k\,\delta\varphi + \frac{k_4}{V_\beta}\beta + \frac{k_3}{V_\beta}\beta'$$

$$\Delta S_\psi'' = l\,\delta V_\varphi - m\,\delta r + n\,\delta V_\alpha - \frac{k_6}{V_\psi \sin\gamma}\Delta\psi - \frac{k_5}{V_\psi \sin\gamma}\Delta\psi'$$

At the same time, it is easy to see that

$$V_\varphi \delta V_\varphi = \delta\varphi',\ \delta V_\alpha = \Delta S_\beta',\ \Delta S_\psi = \delta r,\ \Delta S_\varphi' = \delta V_\varphi,\ \delta V_r = \delta r'$$

The factors a - n are the partial derivations from the sub-integral expressions of the first terms in equations (2.4) - (2.6)

$$a = \frac{V_r}{r},\quad b = \frac{V_\varphi}{r},\quad c = \frac{V_\varphi V_r + V_\alpha^2 \operatorname{tg}\varphi}{r^2},\quad d = \frac{2V_\alpha}{r}\operatorname{tg}\varphi$$

$$e = \frac{V_\alpha^2}{r\cos^2\varphi},\quad f = \frac{V_\alpha}{r}\operatorname{tg}\varphi,\quad g = \frac{V_\alpha}{r},\quad h = \frac{V_\alpha (V_r - V_\varphi \operatorname{tg}\varphi)}{r^2}$$

$$j = \frac{V_r}{r} - \frac{V_\varphi}{r}\operatorname{tg}\varphi,\quad k = \frac{V_\varphi V_\alpha}{r\cos^2\varphi},\quad l = \frac{2V_\varphi}{r},\quad m = \frac{V_\varphi^2 + V_\alpha^2}{r^2},\quad n = \frac{2V_\alpha}{r}$$

For a limited interval of time, these factors can be considered as constants. One should note that this should be correct even for an interval of time which exceeds the period of oscillations of the system, which depends, as is visible from the preceeding, upon the values of factors k_1 - k_6.

In such a manner; we finally have systems of eleven equations with eleven unknowns.

Reducing the order of the characteristic determinant to the sixth one, that is, excluding the five unknowns δV_φ, δV_α, δV_r, $\delta\varphi$, and δr, we will obtain

$$\begin{vmatrix} V_\varphi & 0 & 0 & 1 & 0 & 0 \\ 0 & V_\beta & 0 & 0 & 1 & 0 \\ 0 & 0 & -V_\psi \sin\gamma & 0 & 0 & 1 \\ p^2 - ap - eV_\varphi & -dp & c - bp & -\frac{1}{V_\varphi}(k_1 p + k_2) & -\frac{1}{V_\beta}(k_3 p + k_4) & 0 \\ fp + kV_\varphi & p^2 - jp & h - gp & 0 & 0 & \frac{1}{V_\psi \sin\gamma}(k_5 p + k_6) \\ -lp & -np & p^2 + m & 0 & 0 & 0 \end{vmatrix} = 0$$

Here p -- is the differential operator.

Evaluating this determinant and substituting in the coefficients of the characteristic polynomial for the above accepted numerical values of the magnitudes included in these coefficients, and applying to this polynomial the criterion of Raut and Hurvitz, we will easily deduce that the system is stable because all inequalities of the criterion are satisfied.

Hence, it is apparent that the system is stable (the errors converge to zero).

5. Readings of the system. Direct readings, as is easily seen, can be taken only from readings of the first platform. If $\Delta\varphi$ and β are equal to zero, then the angles of shift of a platform relative to the gyro-system directly yield φ and α .

Regarding the third coordinate r, it is obtained as a result of double integration of the velocities and of the angle $\Delta\psi$, which is multiplied by the current value $1/\gamma_{\psi}\sin\gamma$.

It is natural that the velocities α' , φ' , and r', are obtained after first integration of the velocities and angles β , $\Delta\varphi$, and $\Delta\psi \sin\gamma$ whereupon in order to obtain α' and φ', it is necessary to divide the first integrals by r. Velocities, produced by non-field forces, are obtained directly from the accelerometers.

Velocities, produced by gravitational fields, may be determined indirectly in accordance to the values of the angles $\Delta\varphi$, β , and $\Delta\psi$. Besides this, one should note that a possible system, in which is performed only integration of the angles of deviation $\Delta\varphi$, β, and $\Delta\psi$, but the readings of the accelerometers are not integrated. In such a system, $\Delta\varphi$, β , and $\Delta\psi$ will be characterized by the total velocity, created by field as well as nonfield forces. For extraction from the complete reading of velocity, produced by non-field forces, it is necessary, similar to the foregoing, to utilize readings of accelerometers.

As has already been indicated above, it is possible to employ another system of coordinates for the system. In those cases, when a system of coordinates is applied, differing from the considered one, obviously a more complex system of computer devices is necessary, the structure of which depends upon the method of the accelerometers' attachment in the vehicle. Here, a wide variety is also possible. For example, accelerometers can be attached rigidly to the gyro-platform or directly to the vehicle. It is natural that the acquisition of the necessary coordinates requires the solution of the corresponding functions before integration or after the first integration. Determination of the optimum variant of a system of coordinates and of the method of attachment of the accelerometers from the point of view of the simplification of its instrumentation is a problem which has to be solved in the near future.

Conclusion. 1. The application of gyrosystems during movement in interplanetary space is complicated because of the "weightlessness" of the sensitive elements of the accelerometers and by the absence as a consequence of this correction which produces a period of stability.

2. The method of double integration of error between the reading of the system, which is converted to the position of the optical system, bypasses the indicated difficulty in paragraph 1 and makes possible the determination of movement parameters in space.

3. The indicated method enables obtaining the period of a system which is significantly less than the period of stability and introduces into the system damping without which it would be disturbed by the velocities as occur in geosystems.

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FIGURES

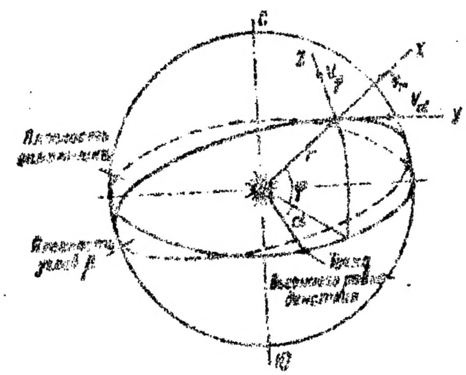

Figure 1.

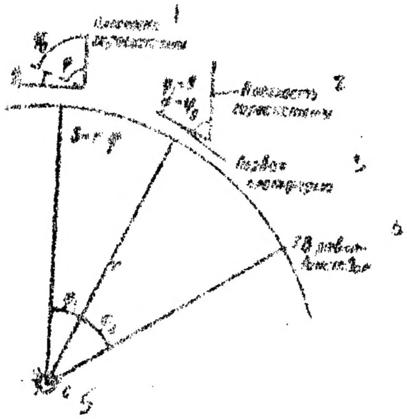

Figure 2.

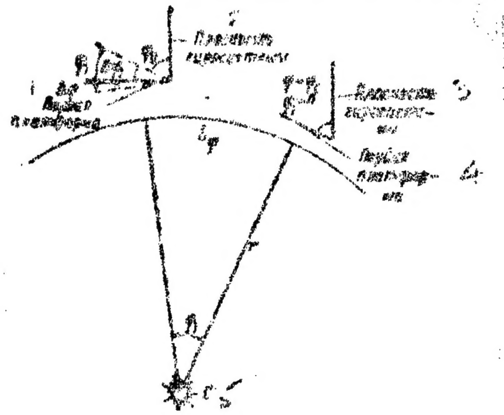

Figure 3.

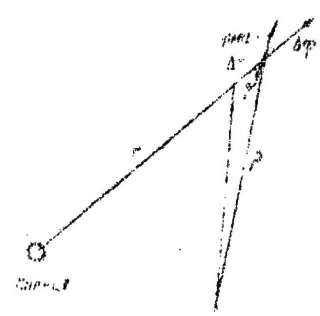

Figure 4.

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